AD A094060

14

ARL-TR-80-22

LEVEL II

Copy No. 66

SOME SIMPLE PROPAGATION MODELS FOR LINEAR AND PARAMETRIC SOURCES IN SHALLOW WATER

T. G. Muir

APPLIED RESEARCH LABORATORIES
THE UNIVERSITY OF TEXAS AT AUSTIN
POST OFFICE BOX 8029, AUSTIN, TEXAS 78712

DTIC
ELECTE
JAN 23 1981
E

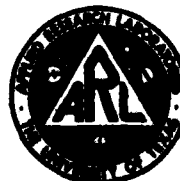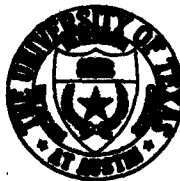9 April 1980

Technical Report

SOME SIMPLE PROPAGATION MODELS FOR LINEAR
AND PARAMETRIC SOURCES IN SHALLOW WATER

by

T. G. Muir

APPLIED RESEARCH LABORATORIES
THE UNIVERSITY OF TEXAS AT AUSTIN
AUSTIN, TEXAS 78712

SOME SIMPLE PROPAGATION MODELS FOR LINEAR
AND PARAMETRIC SOURCES IN SHALLOW WATER

T. G. Muir

Applied Research Laboratories
The University of Texas at Austin
Austin, Texas 78712 U.S.A.

Abstract

Existing theory is extended to model the average field of sound propagating from directive sources in a shallow water waveguide. The role of narrow beam sources in the excitation of modes is explored. Sample calculations are given.

Introduction

In several papers, Weston[1,2] has advanced a simple energy flux method, first suggested by P. J. Westervelt, for the purpose of understanding and giving perspective to several characteristic regimes of shallow water propagation. Although this method is based on elementary geometric considerations, the work of Brekhovskikh[3] may be cited for precise equivalents, obtained from first principles, which support many of the same conclusions. Recently, Shang Er-chang[4] has used a different approach but has arrived at similar results. All of these models are applicable to linear radiations from omnidirectional sources.

The present paper extends Weston's method to include finite beamwidth sources typical of both conventional linear and parametric sources operating in isothermal water. Although the results obtained deal only with the average field intensity as predicted from consideration of generic energy flow and loss mechanisms, they are nonetheless quite useful, both as instructive tools and as simple models for engineering design work.

The incorporation of parametric theory in the present paper is quite flexible in that practically any model that accounts for parametric generation in the freefield provides a viable starting point to the theoretical framework.

Linear Source Models

A shallow water medium of depth H overlays and includes an absorptive sedimentary bottom, having a critical grazing angle ϕ_c that defines a limit for "total" internal reflection. As always, $\phi_c = \arccos c_w/c_s$, where c_w and c_s are the mean sound velocities in the water and sediment media, respectively. The water is assumed to be shallow in the acoustic sense, that is, the ratio H/λ, where λ is the wavelength, is less than a few decades.

The sound source is aimed horizontally from a position S near middepth, and has a halfpower beamwidth of $2\phi_{\frac{1}{2}}$. This source has a nearfield extending to a range $R_0 = \pi a^2/\lambda$, where a is an effective source dimension. For the purpose at hand, it is not necessary to model the transducer nearfield since an extrapolation to a 1 m source level will suffice.

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

We begin with the sonar equation for the sound pressure level at range R_o, in the form,

$$SPL = 10 \log P + 170.8 + DI - 10 \log R_o^2 , \qquad (1)$$

where P is the acoustic power radiated, and DI is the directivity index of the transducer. Some may prefer to set $R_o = 1$ m in these equations; this would make no difference at long ranges.

At ranges $R > R_o$, the radiation proceeds initially into a region of spherical spreading, where the propagation loss becomes

$$L = 10 \log R^2/R_o^2 + \alpha_w R , \qquad (2)$$

Here, α_w denotes attenuation in the water column. The extent of the spherical spreading region may depend on several conditions.

For fairly large source beamwidths, spherical spreading may apply only out to the range associated with the critical angle of the bottom. Thus, for $\phi_{1/2} > \phi_c$, where $\phi_{1/2}$ is the halfpower angle of the beam, we take the extent of this region as $R_o < R < H/2\phi_c$. This simply means that energy has a tendency to be radiated into the bottom at high grazing angles. The geometry for large beam insonification is sketched in Fig. 1.
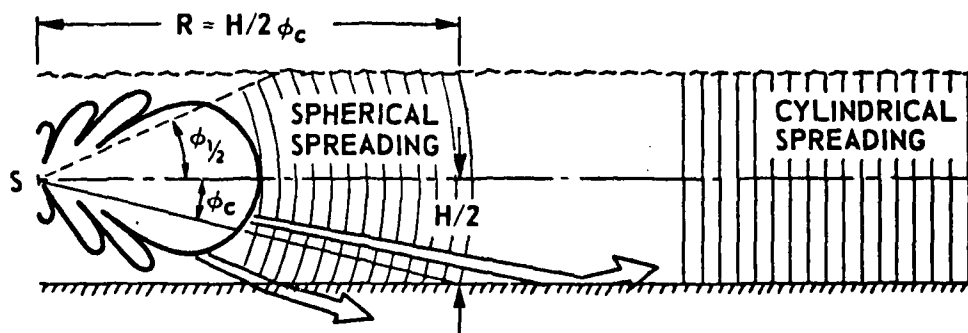

FIGURE 1. LARGE BEAM INSONIFICATION
SHOWING SPHERICAL AND CYLINDRICAL REGIMES

For smaller source beamwidths, such that $\phi_{1/2} < \phi_c$, spherical spreading applies out to the range at which the beam effectively "fills" the water column, i.e., over the span $R_o < R < H/2\phi_{1/2}$. The geometry for this case is sketched in Fig. 2.

Some Simple Propagation Models for Linear and Parametric Sources
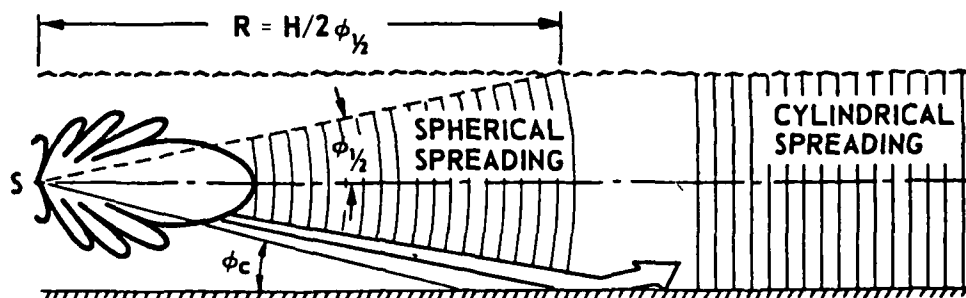in Shallow Water

FIGURE 2. MODERATE BEAM INSONIFICATION
SHOWING SPHERICAL AND CYLINDRICAL REGIMES

Beyond the region of spherical spreading, we enter one of cylindrical
spreading, brought on by increased bottom reflectivity. Provided we are
above the cutoff frequency ($= c_w/4H\phi_c$), the propagation loss becomes

$$L = 10 \log \left[(RH/2\phi)/R_0^2\right] + \alpha_w R \quad , \tag{3}$$

where $\phi = \phi_c$ or $\phi_{\frac{1}{2}}$, whichever is smaller.

For large beams, the cylindrical spreading region is usually quite limited
in lateral extent and in some cases is insignificantly small. It will be shown
that decreasing the source beamwidth increases the extent of this region.
Normally, this region is characterized by the internal reflection of a large
number of rays or modes. The finite attenuation of either conceptual entity
upon bottom reflection ultimately leads to the next propagation regime, known
as the mode stripping region.

Although the source emits a continuum of rays, the medium imposes a
selection process upon their existence, much as a stringed instrument selects
and resonates preferred harmonics of the notes played. In the case of shallow
water propagation the analogous "harmonics" are the spatially selected modes,
each of which consists of a pair of upgoing and downgoing waves ricocheting
down the waveguide on unique angular paths.

Following Weston's approach,[1] we assume that for any allowed modal or
eigenray angle ϕ_i, there is one loss producing bottom bounce each time the
horizontal range cycles through an incremental distance $2H/\phi$. Thus at
range R, there are $n = R\phi/2H$ bottom bounces. If the loss at each such event is
taken as $\alpha\phi$ (in units of dB/rad.), then the bottom loss accumulates as

$$L_B(\phi,R) = \alpha\phi^2 R/2H = 10 \log \exp - \left| \alpha\phi^2 R/8.68H \right| \quad . \tag{4}$$

Weston[2] argued that the gaussian dependence on ϕ in the preceding equation.
should be tied in some way to the critical angle ϕ_c. This was done by allow-
ing the exponent to take on the value $\pi/4$ at $\phi = \phi_c$, which provides for a
little more than 3 dB angle dependent bottom loss at the range $R = 6.8H/\alpha\phi_c^2$.

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

The propagation loss in this region is found by substituting the resulting
value for $\phi_c = \phi = \sqrt{6.8H/\alpha R}$ into Eq. (3), to yield

$$L = 10 \log\left[R^{3/2} H^{1/2} \alpha^{1/2} / 5.2R_o^2\right] + \alpha_w R \quad . \tag{5}$$

This is a form of the well known three halves law appropriate for what is
known as the mode stripping region. Thus, at ranges in the vicinity of $6.8H/\alpha\phi_c^2$,
bottom losses have begun to strip off the modes through a gaussian absorption
factor which is dependent on the steepness of the eigenray angles. This start-
ing range for the applicability of Eq. (5) is of course applicable to the broad
beam case, $\phi_{1/2} > \phi_c$. Although it is difficult to illustrate mode stripping,
a sketch of a few modal eigenrays is offered in Fig. 3 for the case of insoni-
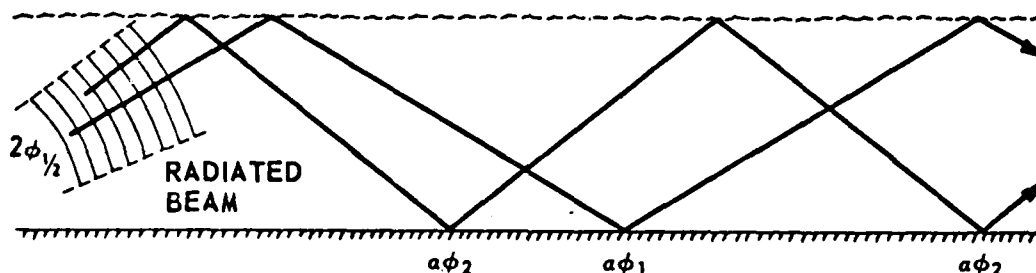fication in a fairly narrow angular sector.

FIGURE 3. MODE STRIPPING FOR THE SMALL BEAMWIDTH CASE
SHOWING ONE EIGENRAY COMPLEMENT FOR TWO MODES DEPICTED

A similar criterion may be applied to the narrow beam case, $\phi_{1/2} \lesssim \phi_c$.
Here, we allow the exponent in Eq. (4) to take on the value $\pi/4$ at the angle
$\phi = \phi_{1/2}$, which provides for a little more than 3 dB angle dependent bottom loss
at the range $R = 6.8H/\alpha\phi_{1/2}^2$. The resulting propagation loss is identical to
Eq. (5). The only difference between the two cases is the range at which
Eq. (5) might become applicable.

Since the stripping of the higher ordered modes is accentuated by the
steepness of their eigenray angles, these modes are eventually attenuated,
leaving only one propagating mode. By the time this has occurred, the surviv-
ing mode will itself have suffered some angle dependent attenuation. Although
the sole surviving mode is sometimes not that of lowest order (depending for
example on bottom peculiarities), we here assume that it is the first mode,
again in consonance with Weston's analysis.[1]

For many real bottoms with phase shifts near π radians, the modal angle
for the first mode is $\phi_1 = \lambda/2H$. Taking this as the angle at which about half
of the energy of the first mode is itself stripped off, we can again use the
exponent of Eq. (4) to determine the range at which this occurs. For broad
beams, the result is $R = 27H^3/\alpha\lambda^2$. Beyond this range, the surface and the
bottom try to contain a sound field propagating at a constant angle, and we are

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

back to cylindrical spreading plus some attenuation losses. The adaptation
of Eq. (3) to this case leads to

$$L = 10 \log \left[(RH/2\phi_1)/R_o^2 \right] + n\alpha\phi_1 + \alpha_w R \quad ,$$

or (6)

$$L = 10 \log RH^2/\lambda R_o^2 + R\lambda^2\alpha/8H^3 + \alpha_w R \quad ,$$

where the second term represents the bottom loss of the first mode.

In the event that one has a very narrow beam, i.e., $\phi_{\frac{1}{2}} \gtrsim \phi_1$, the starting
range of the single mode region is not determined as above. Obviously, in
order to get such a highly directive radiation to propagate, it would be
necessary to aim it at one of the eigenray angles appropriate for the exci-
tation of some mode. This case is illustrated in Fig. 4. In practice, it is
only necessary to employ one beam in the vicinity of an eigenray angle, rather
than the two beams shown. For the first mode, this beam may encompass both
eigenray angles. If this is done, it is reasonable to assume that modal
propagation would begin after the beam has spread to the point of filling up
the water column. In this case, the starting range becomes $R = H/2\phi_{\frac{1}{2}}$.
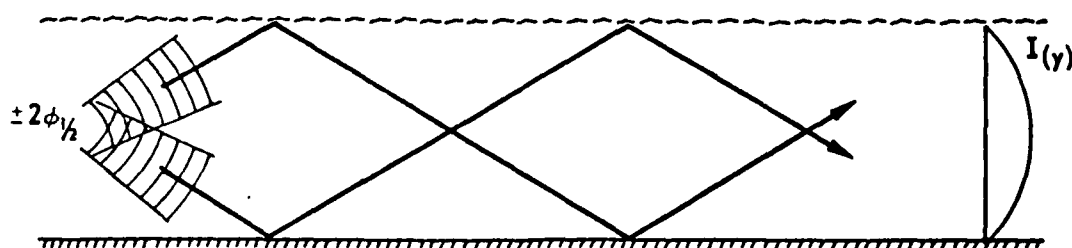
**FIGURE 4. SINGLE MODE EXCITED BY VERY
NARROW BEAMS CENTERED ON EIGENRAY PAIRS**

Although Eqs. (2), (3), (5), and (6) describe successive propagation
regimes, it should be remarked that these may not always follow in succession.
In particular, the use of an extremely narrow beam appears to circumvent some
of these regions. Given that $\phi_{\frac{1}{2}} \approx \lambda/4a$, one can use the inequality defining
the extent of the spherical spreading region to show that it would be skipped
provided $a \gtrsim 2H/\pi$. Similarly, the mode stripping region would be skipped,
provided $\phi_{\frac{1}{2}} \gtrsim \phi_1$.

Parametric Source Models

The application of parametric arrays to shallow water (waveguide) propa-
gation differs in several respects to that of linear sound sources. Physical

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

and engineering aspects of this problem have been discussed in previous papers[5,6]
which have presented theory on preferential mode selection as well as the
results of experiments in both a laboratory tank and in a natural lagoon.

Drawing from previous experience, it is useful to make one restriction
concerning the length of the parametric interaction volume in the shallow
water waveguide. Since there is usually a phase shift near π radians at both
the surface and bottom (at low grazing angles), one must consider the problem
of destructive interference of parametric sources and their images within this
volume. This problem has been treated for the freefield case,[7] and is illus-
trated in this context in Fig. 5. Since 180° phase shifts in each primary
have no effect on the phase of difference frequency generation, it can be
argued that difference frequency sound generated after a phase reversal is
simply 180° out of phase with that reflected from the previous segment of the
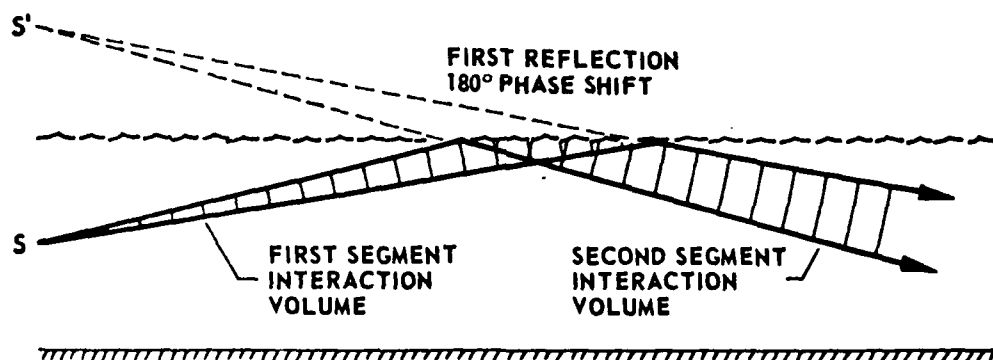interaction volume.

**FIGURE 5. THE PROBLEM OF PHASE REVERSAL
WITHIN A PARAMETRIC INTERACTION VOLUME**

For the case of a shallow water waveguide, it is convenient to avoid the
parametric image interference problem by limiting the effective length of the
interaction volume, L_A. This can be done by design by simply restricting L_A
to distances $\lesssim R_{BF}$, where R_{BF} denotes the range at which the parametric beam
"fills up" the water column. For a source at middepth, $R_{BF} = H/2\phi_{1_2}$. This
case is illustrated in Fig. 6. Since L_A has an exponential taper, the definition
of effective length can be neither exact nor critical. Some might prefer
defining L_A as the 1/e absorption distance of the primaries, i.e., $L_A = 1/\bar{\alpha}_p$,
where $\bar{\alpha}_p$ is the mean primary absorption coefficient. Although this is a safe
and neat definition, it is probably overly restrictive, since there is always
some bottom bounce loss which serves to further truncate the parametric array
in much the same way as does the water column absorption. Perhaps a better
choice is the -3 dB absorption range $L_A = -(\ell n\ 1/\sqrt{2})/\bar{\alpha}_p$. When this is equated
to R_{BF} one obtains $\bar{\alpha}_p \gtrsim -2\phi_{1_2}(\ell n\ 1/\sqrt{2})/H$, which enables the primary frequency f_p
to be determined through $\bar{\alpha}_p = \bar{\alpha}_p(f_p)$. In practice, it may be convenient to

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

truncate the parametric array with finite amplitude attenuation. This enables
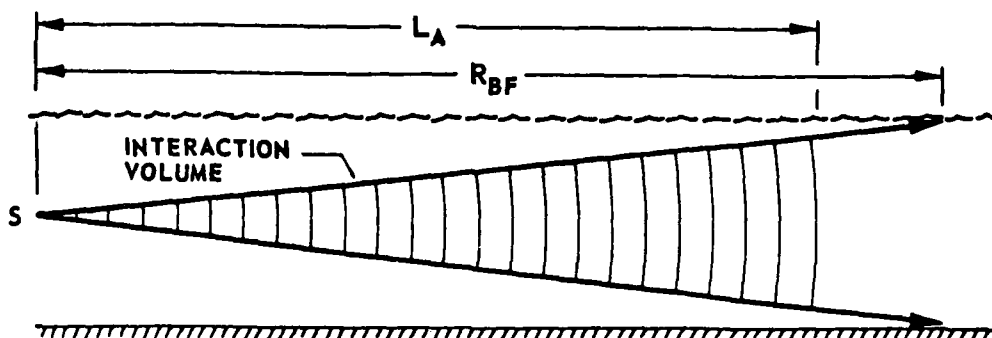one to vary the array length by varying the transmitted power.

**FIGURE 6. RESTRICTION OF PARAMETRIC ARRAY
LENGTHS TO AVOID PHASE REVERSAL INTERFERENCE**

Many theoretical models of parametric arrays operating in infinite media
have been published. The complexity of these solutions usually increases with
their generality and applicability to various problems. Given that the pur-
pose of the present paper is to address shallow water applications with simple
models, we shall favor the simplest parametric array descriptions.

Most parametric array models have their roots in the solution to an in-
homogeneous wave equation[8] which can be written in terms of the difference
frequency sound pressure level as

$$\text{SPL}_d\left(|\vec{R} - \vec{r}|,t\right) = 20 \, \log \left\{ \frac{-\rho_o}{4\pi} \int_V \frac{\partial q}{\partial t} \left[\frac{e^{ik_d|\vec{R} - \vec{r}|}}{|\vec{R} - \vec{r}|} \right] dV \right\} \quad , \qquad (7)$$

Where [·] is the Green's function, $\vec{R}$ is a vector from the origin to the field
point, $\vec{r}$ is a vector from the origin to the element of integration, V denotes
the interaction volume, and q is the source strength density,

$$q(r,t) = \frac{\beta}{\rho_o^2 c_o^4} \frac{\partial}{\partial t} p_p^2 (r,t) \quad . \qquad (8)$$

Here $\beta = 1 + \frac{1}{2}(B/A)$, where B/A is the parameter of nonlinearity, and ρ_o and c_o
are the mean density and sound speed of the medium.

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

The first, shown in Fig. 7, depicts sound propagating at 500 Hz from a
2.3 m diameter source excited with 80 kW of electrical power, assuming a 50%
transducer efficiency. The water column is 50 m deep, overlying a sand bottom
having a sound velocity 15% higher than water, a critical angle of 30°, and
a bottom loss α of 12.4 dB/rad. The transducer is located near middepth,
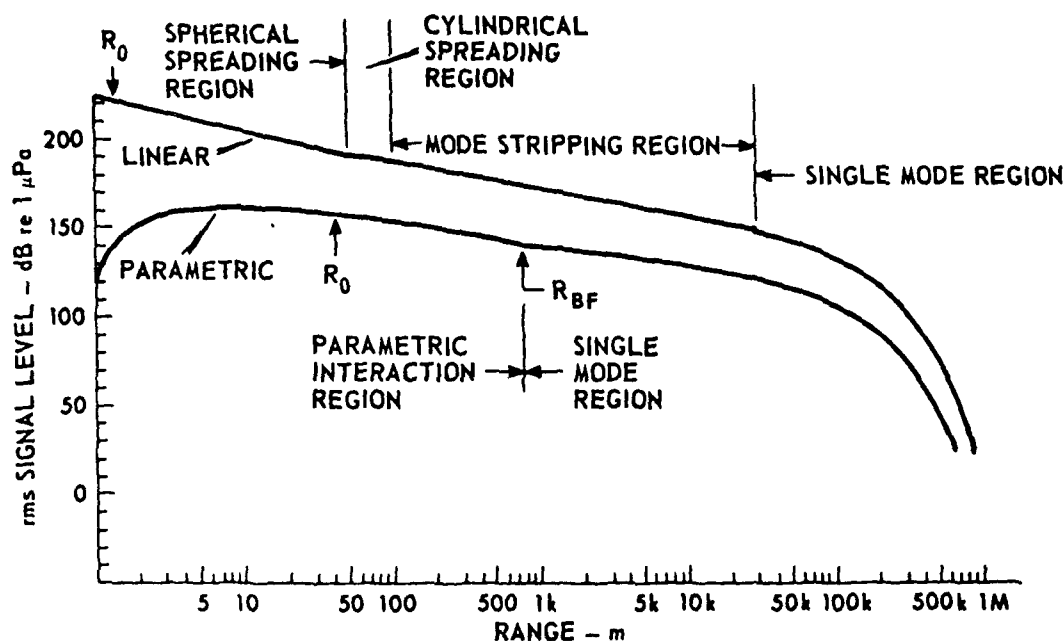with a horizontal orientation.

FIGURE 7. COMPARISON OF MEAN PROPAGATION CURVES FOR A
2.3 m diam SOURCE RADIATING LINEAR AND PARAMETRIC SIGNALS
AT 500 Hz IN A SHALLOW WATER WAVEGUIDE, 50 m DEEP

Since the linear version of this source has a half power beamwidth of
80° (ϕ_{1_2} = 40°), spherical spreading applies out to the critical angle range,
a distance of 48 m. For this case, mode stripping becomes significant at 100 m,
while the single mode region emerges at 31 km. The 3 dB artifact at the
start of this region is discussed in Ref. 1. Although the present models
treat only the mean propagation curve, considerable multipath in the cylin-
drical spreading region and modal interference in the mode stripping region
would be present and would appear as wild fluctuations about the mean, were
they to be taken into consideration.

Parametric operation of this source, also depicted in Fig. 7, arises
from nonlinear interaction of primaries radiated at 13.75 and 14.25 kHz.
Given the same available electrical power and transducer efficiency, one
would here radiate about 20 kW per primary tone. The peak primary source
level during the constructive interference phase of the composite, two tone
radiation extrapolates to 160 dB re 1 μPa at 1 m. Application of weak shock
theory indicates this radiation would form a discontinuity near the Rayleigh

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

distance of 40 m, with stronger shocks occurring in the spherical spreading
field beyond. As a result, this parametric array is truncated by finite ampli-
tude attenuation at a range of about 700 m.

The parametric propagation curve shown in Fig. 7 takes these factors
into consideration, through appropriate solution(s) of Eq. (8). This includes
a three-dimensional numerical integration in the primary nearfield. The far-
field solution was executed by Professor Rolleigh,[14] who reduced the problem
to a single integration which also contains a finite amplitude taper function.[15]
Finite amplitude attenuation broadens the half power angles of the parametric
beam to about ±2°, which leads to beam intersection with the bottom at 735 m.
Since the eigenray angles for the first mode are centered at ±1.7°, most of
the energy in the parametric beam should couple into the first mode of prop-
agation.[5,6] The portion of the curve beyond the array truncation and/or beam
intersection point is therefore constructed from Eq. (7), the single mode
result of the present model. Preferential excitation of the first mode
under these conditions minimizes multipath and multimode interference, and
should make the mean propagation curve depicted here a good representation of
the actual situation.

Although discrete modal excitation is an interesting phenomenon that has
potential as a research tool as well as for clear channel communications,
there are situations where it may be necessary to emphasize other aspects of
the application of parametric arrays to shallow water sound. Perhaps the
most important of these situations is the frequent requirement for higher
parametric source levels. This can be achieved by decreasing the ratio of
primary to secondary frequencies. For the 2.3 m transducer in the 50 m wave-
guide at hand, we might not wish to lower the primary frequencies as this
would increase the length of the parametric array which may lead to destructive
interference effects within the interaction volume. A better approach may
be to raise the difference frequency. Choice of a 2 kHz difference frequency,
for example, provides for a downshift ratio of 7, which compares to a value
of 22 for the previous example at 500 Hz.

Propagation curves for a 2 kHz example are shown in Fig. 8, again com-
paring linear and parametric radiations. The half power angle for the linear
source is 10° which is smaller than the 30° critical angle. This is in con-
tradistinction to the previous case for a 500 Hz radiation. This diminishes
the vertical angular extent of steep angle modes and increases the range span
of the cylindrical spreading region. The mode stripping region is also ex-
panded due to the reduction in wavelength and the consequent excitation of
more modes.

Parametric operation of the source again leads to primary beams of half
power angles near 2'. Since the modal angles are now ±n(0.43)°, we should
excite several modes with the parametric beam. However, the parametric beam
is much smaller than its linear counterpart and this further increases the
extent of the cylindrical spreading region. The mode stripping region
for the parametric source is smaller than for the linear case simply because
of the smaller beam and the smaller number of modes available there for strip-
ping. This region ends at the same range for both linear and parametric sources.

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

Within the cylindrical spreading and mode stripping regions, many interference
fluctuations not treated by the present model will exist. These should be
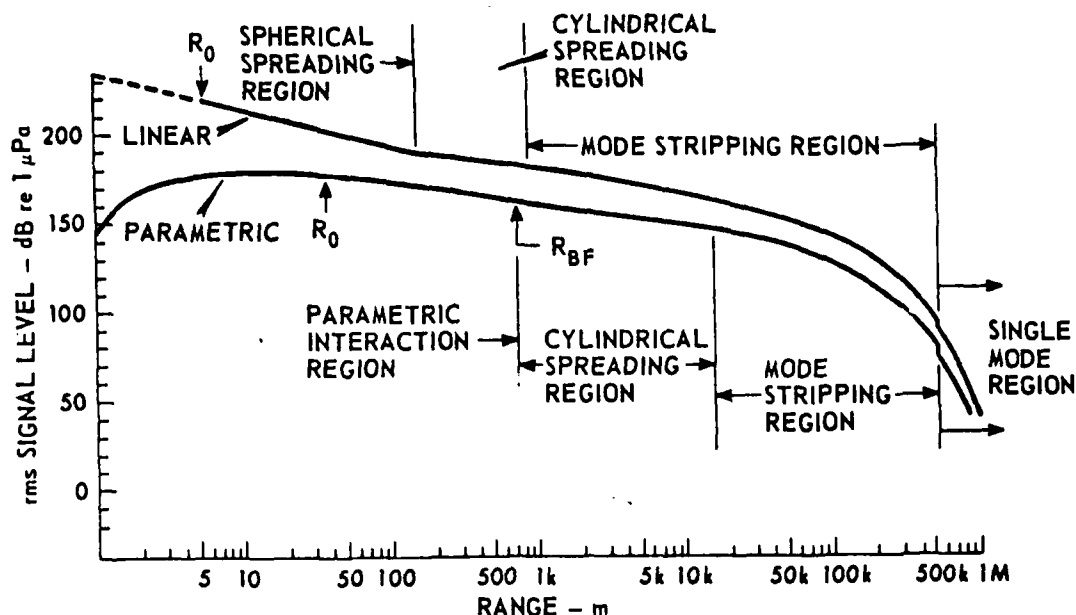less severe for parametric excitation.

FIGURE 8.   COMPARISON OF MEAN PROPAGATION CURVES FOR A
2.3 m diam SOURCE RADIATING LINEAR AND PARAMETRIC SIGNALS
AT 2 kHz IN A SHALLOW WATER WAVEGUIDE, 50 m DEEP

Discussion

Although this paper has addressed only the mean axial propagation char-
acter of linear and parametric sources, many other aspects of the problem
warrant consideration. When the problem of echo to reverberation becomes
important, the high directivity of the parametric source begins to weigh
favorably against equivalent broad beam linear sources. Wide bandwidth capa-
bilities (for signal processing) as well as Doppler considerations[5] also ap-
pear to favor parametric sources.

Vertical strings of linear elements appear to offer better directivity
than the linear array considered here. However, they are usually omnidirec-
tional in the horizontal plane and are usually applied to single mode selection
only as receivers.[16,17]

One should also consider the electroacoustic efficiency of linear trans-
mitting arrays in the very low frequency regime. In the preceding discussion,
an efficiency of 50% was assumed; however, this is usually only possible with
very large resonant elements that would have dimensions of several meters at
500 Hz. Smaller elements have lower efficiencies and this may further favor

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

the practicality of the parametric array for shallow water applications.

Much work remains to be done before these speculations can be evaluated.
The present models are simple and adequate for many purposes but room for
improvement exists. Experimentation is also in its infancy and must of course
be conducted in order to further the understanding of the shallow water problem.

Summary of Results

Parameters:

a = transducer diameter

λ = wavelength

P = acoustic power

DI = directivity index

R = range

$R_o = \pi a^2 / \lambda$ (the Rayleigh distance)

SPL = sound pressure level

ρ_o = static density

k_d = difference frequency wavenumber

$\vec{R}$ = vector from the origin to the field point

$\vec{r}$ = vector from the origin to the element of integration

V = volume of integration

q = source strength density

$\beta = 1 + \frac{1}{2}(B/A)$

where

B/A = parameter of nonlinearity

c_o = mean sound speed

t = time

P_p = primary pressure field

α_w = attenuation coefficient of water

H = water depth

$\phi_{\frac{1}{2}}$ = half angle beamwidth

ϕ_c = critical angle of bottom

α = bottom loss, dB/rad.

L = propagation loss

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

Starting point for a linear radiation at a farfield point $R_o = \pi a^2/\lambda$:

$$SPL = 10 \log P + 170.8 + DI - 10 \log R_o^2 \qquad (1)$$

Starting point for a parametric radiation in the interaction region:

$$SPL_d(R) = 20 \log \left\{ \frac{-\rho_o}{4\pi} \int_V \frac{\partial q}{\partial t} \left[\frac{e^{ik_d |\vec{R} - \vec{r}|}}{|\vec{R} - \vec{r}|} \right] dV \right\} , \qquad (7)$$

where

$$q(r,t) = \frac{\beta}{\rho_o^2 c_o^4} \frac{\partial}{\partial t} p_p^2 (r,t) , \qquad (8)$$

which includes an appropriate finite amplitude taper function for the peak interference maxima in the composite primary pressure $p_p(r,t)$.

Spherical spreading region:

$$L = 10 \log R^2/R_o^2 + \alpha_w R$$

valid for $R_o < R < H/2\phi$ where $\phi = \begin{cases} \phi_c \\ \phi_{\frac{1}{2}} \end{cases}$ whichever is smaller

Cylindrical spreading region:

$$L = 10 \log \left[(RH/2\phi)/R_o^2 \right] + \alpha_w R$$

valid for $H/2\phi < R < 6.8H/\alpha\phi^2$ where $\phi = \begin{cases} \phi_c \\ \phi_{\frac{1}{2}} \end{cases}$ whichever is smaller

Mode stripping region:

$$L = 10 \log \left[R^{\frac{3}{2}} H^{\frac{1}{2}} \alpha^{\frac{1}{2}}/5.2R_o^2 \right] + \alpha_w R$$

valid for $6.8H/\alpha\phi^2 < R < \begin{cases} 27H^3/\alpha\lambda^2 & \text{for } \phi_{\frac{1}{2}} > \phi_1 \\ \\ H/2\phi_{\frac{1}{2}} & \text{for } \phi_{\frac{1}{2}} \gtrsim \phi_1 \end{cases}$ where $\phi = \begin{cases} \phi_c \\ \phi_{\frac{1}{2}} \end{cases}$ whichever is smaller

Some Simple Propagation Models for Linear and Parametric Sources in Shallow Water

Single mode region:

$$L = 10 \log RH^2/\lambda R_o^2 + R\lambda^2 \alpha/8H^3 + \alpha_w R$$

$$\text{valid for } R > \begin{cases} 27H^3/\alpha\lambda^2 & \text{for } \phi_{\frac{1}{2}} > \phi_1 \\ H/2\phi_{\frac{1}{2}} & \text{for } \phi_{\frac{1}{2}} \gtrsim \phi_1 \end{cases}$$

Acknowledgements

References

1. D. E. Weston, "Intensity-Range Relations in Oceanographic Acoustics," J. Sound Vib., 18(2), 271-287 (1971).

2. D. E. Weston, "Shallow Water Propagation," in IEEE EASCON proceedings, 252-255, pub. 78CH (1978).

3. L. M. Brekhovskikh, Waves in Layered Media, (English translation), Academic Press, New York (1960).

4. Shang Er-chang, "Transition Ranges of the Average Sound Field in Shallow Water," Scientia Sinica, XIX, 794-804 (1976).

5. T. G. Muir and J. R. Clynch, "Propagation of Parametric Waves in Shallow Water," proceedings of the conference Recent Developments in Underwater Acoustics, Institute of Acoustics meeting at the Admiralty Underwater Weapons Establishment, Portland, England, April 1976.

6. J. R. Clynch and T. G. Muir, "Shallow Water Parametric Arrays," published proceedings abstract and unpublished paper, Seventh International Symposium on Nonlinear Acoustics, Virginia Polytechnic Institute, August 1976.

7. T. G. Muir, L. L. Mellenbruch, and J. C. Lockwood, "Reflections of finite-amplitude waves in a parametric array," J. Acoust. Soc. Am. 62, 271-276 (1977).

Some Simple Propagation Models for Linear and Parametric Sources
in Shallow Water

8. P. J. Westervelt, "Parametric Acoustic Array," J. Acoust. Soc. Am. 35, 535-537 (1963).

9. T. G. Muir and J. G. Willette, "Parametric Acoustic Transmitting Arrays," J. Acoust. Soc. Am. 52, 1481-1486 (1972).

10. R. T. Beyer, Nonlinear Acoustics, U.S. Government Printing Office (1974).

11. See, for example, Zarembowitch, ed., "Proceedings of the 8th ISNA, Paris, 1978," J. de Physique (in publication).

12. H. O. Berktay and D. J. Leahe, "Farfield performance of parametric transmitters," J. Acoust. Soc. Am. 55, 539-546 (1974).

13. See, for example, J. C. Lockwood, "Investigation of Finite Amplitude Attenuation," AMETEK Technical Report No. 11-1554E-76-6, Ametek-Straza Corp., El Cajon, California (1976).

14. R. L. Rolleigh, private communication. See also, R. L. Rolleigh, "Difference frequency pressure within the interaction region of a parametric array," J. Acoust. Soc. Am. 58, 964-971 (1975).

15. R. H. Mellen and M. B. Moffett, "A Model for Parametric Sonar Radiator Design," U. S. Navy Underwater Systems Center Technical Memorandum PA41-229-71, New London, Connecticut.

16. R. H. Ferris, "Comparison of Measured and Calculated Normal-Mode Amplitude Functions for Acoustic Waves in Shallow Water," J. Acoust. Soc. Am. 52, 981-988 (1972).

17. Shang Er-chang, private communication of the preprint, "The Potential Capability of Suppression of Near Point Interference Sources by Using a Vertical Mode Selective Array," Institute of Acoustics, Chinese Academy of Science, Peking (1978).

DISTRIBUTION LIST FOR
ARL-TR-80-19, ARL-TR-80-22, and ARL-TR-80-23
UNDER CONTRACT N00014-75-C-0161

Copy No.

	Office of Naval Research
	Department of the Navy
	Arlington, VA 22217
1 - 2	Attn: R. Obrochta, Code 464
3	CAPT W. Boyer
4	CDR E. O'Brien, Code 200
5	R. Winokur, Code 102C

	Commander
	Naval Sea Systems Command
	Department of the Navy
	Washington, DC 20362
6	Attn: E. Liszka, Code 63R1
7	D. Porter, Code 63R3
8	F. Romano, Code 63R3
9	A. W. Bradley, Code 63X14
10	CAPT P. Wright, Code PMS 407
11	CDR J. Post, Code PMS 407
12	C. Taylor, Code 05H3
13	N. G. Jenkins, Code 63X32

	Commander
	Naval Material Command
	Department of the Navy
	Washington, DC 20360
14	Attn: CAPT O'Keefe, Code 08TB
15	T. Horwath, Code 08TC
16	CDR Young, Code 08D13
17	G. Spaulding, Code 08T23
18	J. Probus, Code 08L

	Commander
	Naval Air Systems Command
	Washington, DC 20361
19	Attn: W. Emshwiller, Code 548C2
20	D. Rosso, Code 370

	Commanding Officer
	Naval Electronic Systems Command
	Department of the Navy
	Washington, DC 20360
21	Attn: J. Sinsky, Code 320
22	CAPT Cox, Code PME 124

Dist. List for ARL-TR-80-19, ARL-TR-80-22, and ARL-TR-80-23 under
Contract N00014-75-C-0161

Copy No.

	Office of the Chief of Naval Operations Department of the Navy Washington, DC 20350
23	Attn: Capt L. Sykes, Code OP-374
24	D. Bradley, Code OP-374
25	CDR Hendrickson, Code OP-981D
26	CAPT Keathley, Code OP-954
27	CAPT W. Christensen, Code OP-951F
28	CDR Kelley
29	CAPT J. Van Metre, Code OP-224
30	LCDR J. Carter, Code OP-961C4
31	CDR Harlett, Code OP-952D1

	Director Naval Research Laboratory Department of the Navy Washington, DC 20375
32	Attn: J. Munson, Code 5100
33	F. Ingenito, Code 5120
34	W. Newbauer, Code 5108
35	L. Dragonette

	Commander Naval Coastal Systems Center Department of the Navy Panama City, FL 32407
36	Attn: H. Warner, Code 791
37	D. Skinner, Code 791
38	M. Lacey, Code 720
39	J. Hammond, Code 722
40	D. Folds

	Officer in Charge New London Laboratory Naval Underwater Systems Center Department of the Navy New London, CT 06320
41	Attn: M. Moffett
42	R. H. Mellen
43	W. Konrad

	Commanding Officer Naval Surface Weapons Center White Oak Laboratory Department of the Navy Silver Spring, MD 20910
44	Attn: G. Gaunard, Code R31
45	G. Kalaf, Code U5

Dist. List for ARL-TR-80-19, ARL-TR-80-22, and ARL-TR-80-23 under
Contract N00014-75-C-0161

Copy No.

	Commander Naval Ocean Systems Center Department of the Navy San Diego, CA 92152
46	Attn: H. Bucher, Code 5311
47	F. Gordon, Code 633
48	J. Reeves, Code 635
49	W. Angeloff, Code 6233

Commanding Officer
Naval Ocean Research and Development Activity
NSTL Station, MS 39529

50   Attn: R. Goodman, Code 110
51   S. Marshal, Code 340
52   J. Posey, Code 340
53   Mr. Soloman, Code 500

Defense Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, VA 22209

54   Attn: T. Kooij, TTO
55   CDR V. Simmons

56   Commander
            Naval Intelligence Command
            2461 Eisenhower Avenue
            Alexandria, VA 22331

57   Commander
            Naval Explosive Ordnance Disposal Facility
            Indian Head, MO 20640
            Attn: Mr. L. H. Dickinson, Technical Director

58   Naval Research Laboratory
            Underwater Sound Reference Division
            Orlando, FL 32806
            Attn: J. Blue

59   Officer-in-Charge
            David W. Taylor Naval Ship Research and Development Center
            Department of the Navy
            Bethesda, MD 20084
            Attn: R. Rippeon

60   National Academy of Sciences
            Naval Studies Board
            2101 Constitution Avenue, NW
            Washington, DC 20418
            Attn: L. Hunt

Dist. List for ARL-TR-80-19, ARL-TR-80-22, and ARL-TR-80-23 under
Contract N00014-75-C-0161

Copy No.

	North Atlantic Treaty Organization
	SACLANT ASW Research Centre
	APO 09019
	New York, NY
61	Attn: W. Kuperman
62	F. Whicker
63	F. Jensen

64   The Catholic University of America
Washington, DC 20064
Attn: Dr. Überall

65   Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16801
Attn: F. H. Fenlon

66 - 67   Commanding Officer and Director
Defense Technical Information Center
Cameron Station, Building 5
5010 Duke Street
Alexandria, VA 22314

68   Office of Naval Research
Resident Representative
Room No. 582, Federal Bldg.
Austin, TX 78701

69   Library, ARL:UT

70 - 79   Reserve, ARL:UT